# 通量计数中的坐标旋转修正：二次坐标旋转与平面拟合旋转

周新华

Campbell Scientific, US

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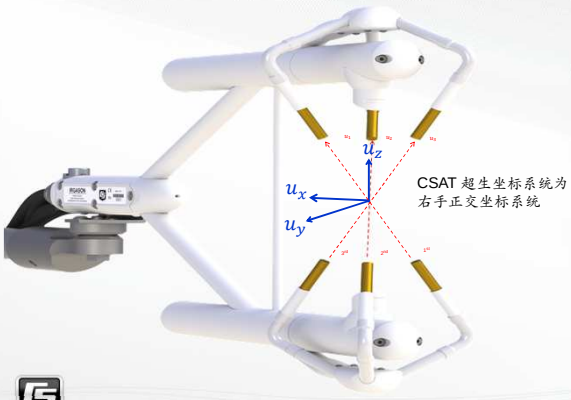
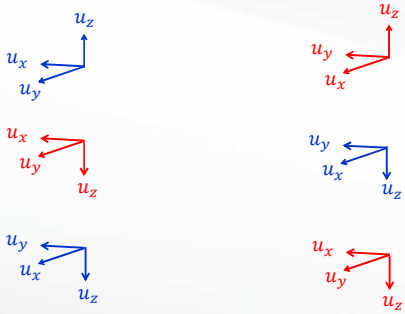
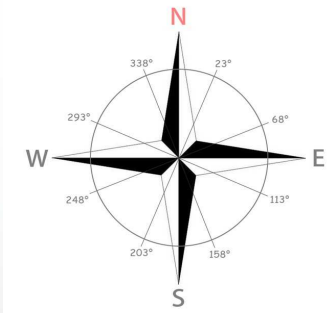


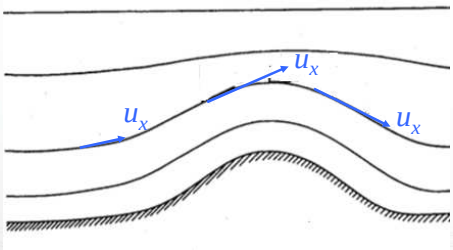

CSAT 超生坐标系统为右手正交坐标系统

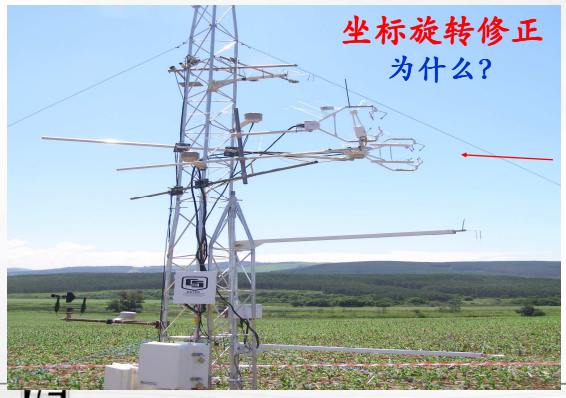
## 左手正交坐标系与右手坐标系

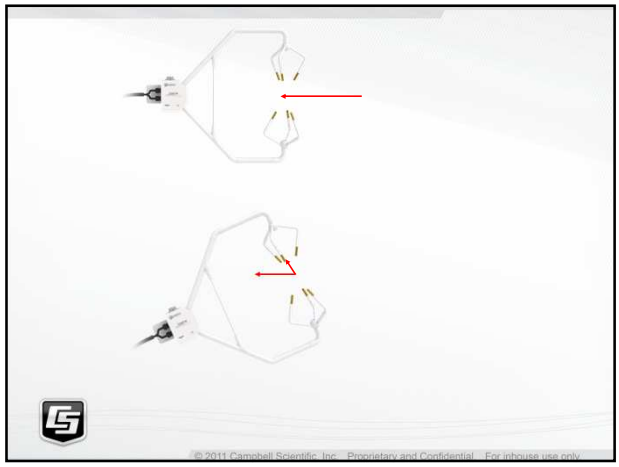


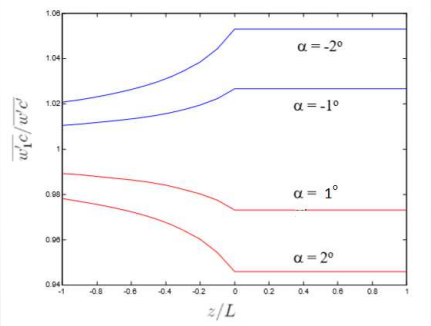



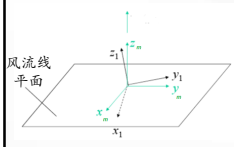

## 自然风坐标系统







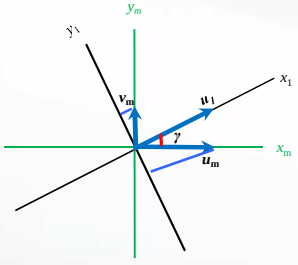

$(x_m, y_m, z_m)$ 风速仪坐标系

$(x_1, y_1, z_1)$ 自然风坐标系

## 通量计算中的坐标旋转

将超声风速仪测得并表达在其右手正交坐标系的三维风表达到自然风坐标系中。

实际上是将由风速仪坐标系的动量通量，感热通量，$CO_2$/$H_2O$/痕量气体通量表达到自然风坐标系。

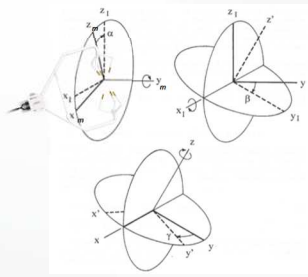



$$u_1 = u_m \cos\gamma + v_m \sin\gamma$$

$$v_1 = -u_m \sin\gamma + v_m \cos\gamma$$

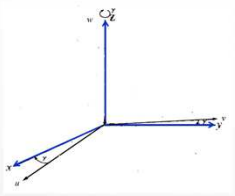
$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} u_m \\ v_m \end{bmatrix}$$

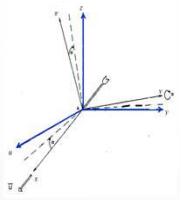
## 三步坐标旋转示意



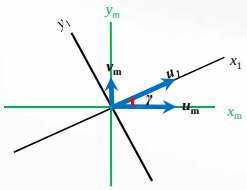
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} u_m \\ v_m \\ w_m \end{bmatrix}$$

## 二次坐标旋转

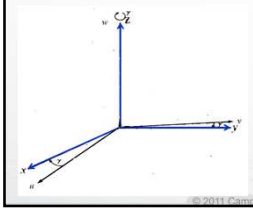




## 第一次旋转

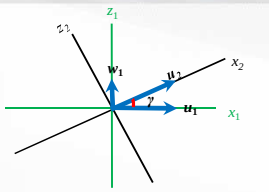


$$\gamma = \arctan\left(\frac{\overline{v}_m}{\overline{u}_m}\right)$$

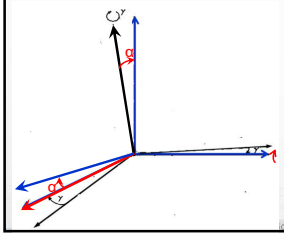


$$\begin{bmatrix}\overline{u}_1\\ \overline{v}_1\\ \overline{w}_1\end{bmatrix} = \begin{bmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix}$$

## 第二次旋转

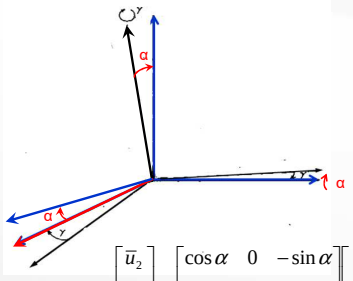


$$\alpha = -\arctan\frac{\overline{w}_1}{\overline{u}_1} = -\arctan\frac{\overline{w}_m}{\overline{u}_m\cos\gamma - \overline{v}_m\sin\gamma}$$



$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & -\sin\alpha\\ 0 & 1 & 0\\ \sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}\overline{u}_1\\ \overline{v}_1\\ \overline{w}_1\end{bmatrix}$$

## 两次旋转



$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & -\sin\alpha\\ 0 & 1 & 0\\ \sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix}$$

## 动量协方差的坐标旋转

$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix} = \begin{bmatrix}\cos\alpha\cos\gamma & \cos\alpha\sin\gamma & -\sin\alpha\\ -\sin\gamma & \cos\gamma & 0\\ \sin\alpha\cos\gamma & \sin\alpha\sin\gamma & \cos\alpha\end{bmatrix}\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix} = \mathbf{R}_2\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix}$$

$$\begin{bmatrix}u_2'\\ v_2'\\ w_2'\end{bmatrix} = \mathbf{R}_2\begin{bmatrix}u_m'\\ v_m'\\ w_m'\end{bmatrix}$$

$$\begin{bmatrix}u_2'\\ v_2'\\ w_2'\end{bmatrix}\begin{bmatrix}u_2' & v_2' & w_2'\end{bmatrix} = \mathbf{R}_2\begin{bmatrix}u_m'\\ v_m'\\ w_m'\end{bmatrix}\begin{bmatrix}u_m' & v_m' & w_m'\end{bmatrix}\mathbf{R}_2^{\mathsf{T}}$$

$$\begin{bmatrix}\overline{u_2'^2} & \overline{u_2'v_2'} & \overline{u_2'w_2'}\\ \overline{u_2'v_2'} & \overline{v_2'^2} & \overline{v_2'w_2'}\\ \overline{u_2'w_2'} & \overline{v_2'w_2'} & \overline{w_2'^2}\end{bmatrix} = \mathbf{R}_2\begin{bmatrix}\overline{u_m'^2} & \overline{u_m'v_m'} & \overline{u_m'w_m'}\\ \overline{u_m'v_m'} & \overline{v_m'^2} & \overline{v_m'w_m'}\\ \overline{u_m'w_m'} & \overline{v_m'w_m'} & \overline{w_m'^2}\end{bmatrix}\mathbf{R}_2^{\mathsf{T}}$$

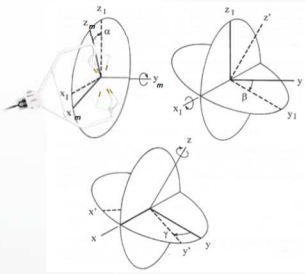
## 标量通量的坐标旋转

$$Q\begin{bmatrix} u_2' \\ v_2' \\ w_2' \end{bmatrix} = \mathbf{R}_2 Q \begin{bmatrix} u_m' \\ v_m' \\ w_m' \end{bmatrix}$$

$$\begin{bmatrix} \overline{Qu_2'} \\ \overline{Qv_2'} \\ \overline{Qw_2'} \end{bmatrix} = \mathbf{R}_2 \begin{bmatrix} \overline{Qu_m'} \\ \overline{Qv_m'} \\ \overline{Qw_m'} \end{bmatrix}$$



## 三步坐标旋转示意



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_m \\ v_m \\ w_m \end{bmatrix}$$



$$\begin{bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \overline{u}_2 \\ \overline{v}_2 \\ \overline{w}_2 \end{bmatrix}$$

$$\begin{bmatrix} u' \\ v' \\ v' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} u_2' \\ v_2' \\ w_2' \end{bmatrix}$$



$$\overline{u'^2} = \overline{u_2'^2}$$

$$\overline{v'^2} = \overline{v_2'^2}\cos\beta + 2\overline{v_2'w_2'}\sin\beta\cos\beta + \overline{w_2'^2}\sin\beta$$

$$\overline{w'^2} = \overline{v_2'^2}\cos\beta - 2\overline{v_2'w_2'}\sin\beta\cos\beta + \overline{w_2'^2}\sin\beta$$

$$\overline{u'v'} = \overline{u'v'}\cos\beta + \overline{u'w'}\sin\beta$$

$$\overline{u'w'} = -\overline{u'v'}\sin\beta + \overline{u'w'}\cos\beta$$

$$\overline{v'w'} = -\frac{1}{2}\sin 2\beta\left(\overline{v_2'^2} - \overline{w_2'^2}\right) + \overline{v_2'w_2'}\cos 2\beta$$



$$\overline{v'w'} = -\left(\overline{v_2'^2} - \overline{w_2'^2}\right)\frac{1}{2}\sin 2\beta + \overline{v_2'w_2'}\cos 2\beta$$

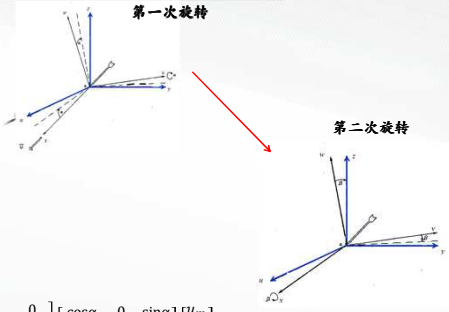
如果 $\overline{v'w'} = 0$

则 $\tan 2\beta = \dfrac{2\overline{v_2'w_2'}}{\overline{v_2'^2} - \overline{w_2'^2}}$

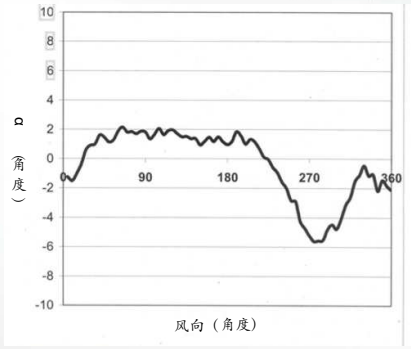
$$\beta = \frac{1}{2}\arctan\left(\frac{2\overline{v_2'w'}_2}{\overline{v_2'^2} - \overline{w_2'^2}}\right)$$



## 平面拟合坐标旋转

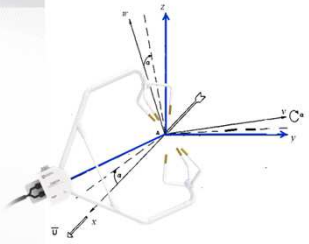



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} u_m \\ v_m \\ w_m \end{bmatrix}$$




Finnigan et al. (2002)



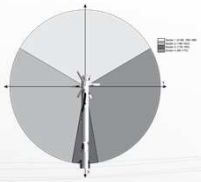


0~60 300~360

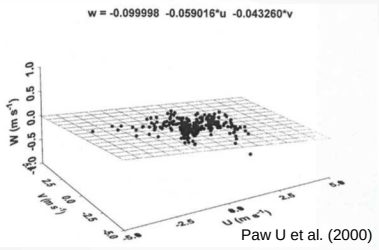
>60 & <=170

>170 & <190

>=190 & <300








Paw U et al. (2000)

$$\tan\alpha = 0.059016$$

$$\tan\beta = -0.043260$$



$$\begin{bmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ \sin\alpha\sin\beta & \cos\beta & \sin\beta\cos\alpha\\ -\sin\alpha\cos\beta & -\sin\beta & \cos\alpha\cos\beta\end{bmatrix}\begin{bmatrix}\bar{u}_m\\ \bar{v}_m\\ \bar{w}_m\end{bmatrix} = \mathbf{R}_p\begin{bmatrix}\bar{u}_m\\ \bar{v}_m\\ \bar{w}_m\end{bmatrix}$$

$$\begin{bmatrix}u'\\ v'\\ w'\end{bmatrix} = \mathbf{R}_p\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix}$$

$$\begin{bmatrix}u'\\ v'\\ w'\end{bmatrix}\begin{bmatrix}u' & v' & w'\end{bmatrix} = \mathbf{R}_p\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix}\begin{bmatrix}u'_m & v'_m & w'_m\end{bmatrix}\mathbf{R}_p^T$$

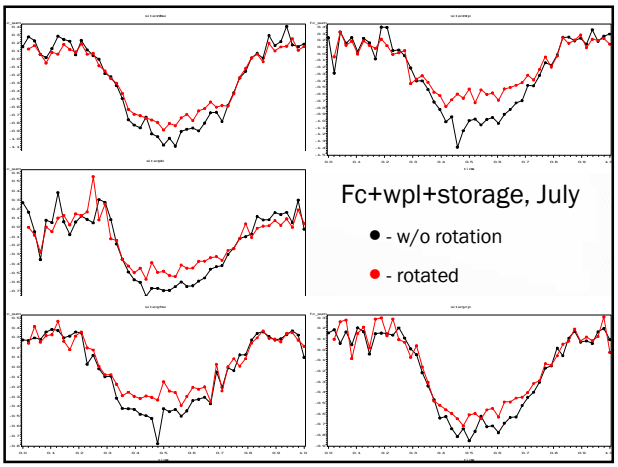


$$\begin{bmatrix}\overline{u'^2} & \overline{u'v'} & \overline{u'w'}\\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'}\\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2}\end{bmatrix} = \mathbf{R}_p\begin{bmatrix}\overline{u'^2_m} & \overline{u'_m v'_m} & \overline{u'_m w'_m}\\ \overline{u'_m v'_m} & \overline{v'^2_m} & \overline{v'_m w'_m}\\ \overline{u'_m w'_m} & \overline{v'_m w'_m} & \overline{w'^2_m}\end{bmatrix}\mathbf{R}_p^T$$



$$Q'\begin{bmatrix}u'\\ v'\\ w'\end{bmatrix} = \mathbf{R}_p Q'\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix}$$

$$\begin{bmatrix}\overline{Q'u'}\\ \overline{Q'v'}\\ \overline{Q'w'}\end{bmatrix} = \mathbf{R}_p\begin{bmatrix}\overline{Q'u'_m}\\ \overline{Q'v'_m}\\ \overline{Q'w'_m}\end{bmatrix}$$




## 二次旋转

$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix}=\begin{bmatrix}\cos\alpha & 0 & -\sin\alpha\\ 0 & 1 & 0\\ \sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix}$$

## 平面拟合

$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\beta & \sin\beta\\ 0 & -\sin\beta & \cos\beta\end{bmatrix}\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}\overline{u}_m\\ \overline{v}_m\\ \overline{w}_m\end{bmatrix}$$



## 主要参考文献

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Wallace, J.M. and P.V. Hobbs. 2006. Atmospheric Science: An introductory Survey.

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## 谢谢!